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ABSORPTION OF IONOSPHERICALLY PROPAGATED HF RADIO WAVES UNDER CONDITIONS WHERE THE QUASI-TRANSVERSE (QT) APPROXIMATION IS VALID

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Prepared for:

U.S. ARMY ELECTRONICS LABORATORIES
FORT MONMOUTH, NEW JERSEY

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ABSTRACT

This report presents expressions for the absorption coefficients of a plane electromagnetic wave propagating through regions of a weakly ionized magneto-ionic medium where the quasi-transverse (QT) approximation is valid. The QT approximation is most likely to hold in the ionosphere on long east-west and short equatorial paths. A comparison of absorption on representative paths calculated by using absorption coefficients for the upper and lower sign shows that the upper sign, the ordinary wave, suffers less attenuation than the lower, the extraordinary wave, and should be used for communicating. The ordinary wave is launched on short equatorial paths by aligning both transmitting and receiving antennas parallel to the earth's magnetic field. When the wave frequency is near the gyrofrequency, both the ordinary and the extraordinary waves are greatly absorbed, because deviative absorption is occurring in a "high-loss" region. When the wave frequency is much greater than the gyrofrequency ~~(about 6 to 10 times)~~ the expression for both signs with the QT approximation is very nearly the same as the expression for the simple no-field case. Sample calculations for *D*-, *E*-, and *F*-region absorption are presented. A brief investigation of full-wave theory near the height of reflection for the vertical-incidence case indicates that integration of the absorption coefficients to $X = 1$ and $X = 1 - Y$ requires an additive correction that depends on the electron- and collision-frequency gradients near these heights as well as on the absolute collision frequency. For the case of the wave frequency relative to the critical frequency (f/f_c) not too near unity, the correction can be as large as several decibels; for $f/f_c = 1$, the correction can be very large indeed.

Experiments could be conducted in Thailand to test the validity of these absorption coefficient expressions calculated for the vertical- and near-vertical-incidence paths by using the magneto-ionic theory. Such measurements would improve the calculation of lowest useful frequency (LUF).

The application of absorption coefficient expressions to LUF calculations is discussed in Appendix B of this report.

PREFACE

This report supersedes Chap. III of Research Memorandum 5 on this contract, "Orientation of Linearly Polarized HF Antennas for Short-Path Communication via the Ionosphere near the Geomagnetic Equator," dated August 1963. While the basic conclusions of that chapter were valid, this report corrects several errors in details and presents much supplementary information, including the application of the absorption coefficients to calculation of lowest useful frequency (LUF).

The expanded text and computations in this report are presented to further understanding of short-distance HF communication in regions near the geomagnetic equator. This report is one of a series of reports investigating various aspects and problems of communication in equatorial regions. Readers interested in over-all aspects of communication in tropical areas are urged to review all the reports of the series.

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I INTRODUCTION

A reasonable model for ionospheric propagation for short ground ranges (less than 50 miles), and neglecting tilts (non-horizontally stratified electron density), near the magnetic equator[1,2,3]* is a planar ionosphere with a horizontal, static, magnetic field. Under these conditions, at quasi-vertical incidence, the propagation is very nearly transverse to the earth's magnetic field, and the quasi-transverse (QT) approximation[4] is likely to hold over some of the useful frequency range.

The wave polarization for the characteristic waves (those propagating in the ionosphere with locally unchanging wave polarization) is linear. It is possible, therefore, to launch a characteristic wave with a simple, linearly polarized antenna, such as a horizontal dipole. Theoretically, either the ordinary or the extraordinary wave could be launched to avoid the polarization fading (*O-X* fading) observed by Singh and Ram[5] and others at frequencies near the MUF, by exciting only one of the characteristic waves (neglecting coupling within the ionosphere). Fading could not be eliminated, however, since it has other causes.[6] Measurements at vertical incidence taken by Busch in Thailand indicate that the *O* and *X* fade relatively independently.[7] Cohen points out that mode coupling is relatively more likely when the QT approximation is about to fail.[8] One would expect such polarization fading to be worse when the QT approximation holds than when the quasi-longitudinal (QL) approximation holds, because differential absorption between *O* and *X* is not as great for QT as for QL (see Sec. V). The differential absorption is significant; it must be considered when the signal strength of the downcoming wave is being maximized for a given transmitter setup. The characteristic wave to be excited is partially indicated by a consideration of the absorption of the ordinary and extraordinary waves in regions where the QT approximation is valid. Such a consideration is also pertinent to the calculation of usable frequencies, especially in the lower part of the HF band. The calculations of Sec. III indicate that the ordinary wave suffers the less absorption. The mathematics required for such calculations are developed in the remainder of this Introduction and in Sec. II.

---

* References are given at the end of the report.

Appleton's magneto-ionic equations—presented before an URSI Meeting in 1927—for refractive index, $n = \mu - i\chi$, and wave polarization, R, are written in Ratcliffe's nomenclature:[4]

$$n^2 = 1 - \cfrac{X}{1 - iZ - \cfrac{Y_T^2}{2(1 - X - iZ)} \pm \left[\cfrac{Y_T^4}{4(1 - X - iZ)^2} + Y_L^2\right]^{1/2}}$$

$$R = \frac{Ex}{Ey} = \left(-\frac{i}{Y_L}\right)\left\{\frac{Y_T^2}{2(1 - X - iZ)} \mp \left[\frac{Y_T^4}{4(1 - X - iZ)^2} + Y_L^2\right]^{1/2}\right\}$$

where*

X = normalized electron plasma frequency squared
θ = angle between wave normal and static magnetic field
Y = normalized gyrofrequency
Y_L = normalized gyrofrequency, longitudinal ($Y \cos \theta$)
Y_T = normalized gyrofrequency, transverse ($Y \sin \theta$)
Z = normalized collision frequency.

The upper sign is, for the HF case, usually referred to as the ordinary wave, that is, the wave least affected by the earth's magnetic field. The lower sign is termed the extraordinary.

The original attempts at simplifying these expressions were made by Henry Booker in the mid-1930's.[9,10] Observing that the quadratic term can be greatly simplified if the inequality

$$\frac{Y_T^4}{4Y_L^2} >> |(1 - X - iZ)^2|$$

is satisfied, Booker derived the following approximations:

$$n^2_{QT\ (upper)} \cong 1 - \frac{X}{1 - iZ}$$

$$R_{QT\ (upper)} \cong 0$$

---

* The subscripts L and T refer to longitudinal and transverse components, respectively, as determined by the angle, θ, between the wave normal and the earth's field at any point along the ray path. The nomenclature is described in more detail in Appendix A of this report.

$$n^2_{QT\,(lower)} \cong 1 - \cfrac{X}{1 - iZ - \cfrac{Y_T^2}{1 - X - iZ}}$$

$$R_{QT\,(lower)} \cong \infty$$

which are the same equations that would apply if the propagation were transverse (*i.e.*, $Y = Y_T$). Observe that the expression for the upper sign corresponds to the refractive index expression when the earth's magnetic field is neglected.

In 1952 Whitehead pointed out that the expression for the upper sign should have another term to be an adequate approximation. An approximation that is valid for the quadratic is not necessarily valid for the entire expression. Whitehead's correction is:

$$n^2_{QT\,(upper)} \cong 1 - \frac{X}{1 - iZ + (1 - X - iZ)\cot^2\theta}$$

$$R_{QT\,(upper)} \cong iY_L\left(\frac{1 - X - iZ}{Y_T^2}\right) \cong 0.$$

This is most easily seen by making a binominal expansion for the quadratic expression, which also modifies the expression for wave polarization, R. Other approximate expressions, useful for calculation, were given by Bailey in 1958.[12]

In 1959 Ratcliffe presented expressions for the QT approximation[4] using the upper-sign binomial expansion of Whitehead and the lower-sign expressions given by Booker in 1935. In 1961 Davies and King presented a binomial expansion expression for the lower sign, but neglected collisions.[13] In their paper they discussed the validity of various approximations, giving data resulting from digital computer programs and using Appleton's full expression as a basis of comparison. In 1962, Hibberd presented the following expression:[14]

$$n^2_{QT\,(lower)} \cong 1 - \cfrac{X}{1 - iZ - \cfrac{Y_T^2}{1 - X - iZ} - (1 - X - iZ)\cot^2\theta}$$

$$R_{QT\ (\mathrm{lower})} \cong -\frac{iY_T^2}{Y_L(1 - X - iZ)} \cong \infty.$$

Hibberd gave many forms for the approximations and strongly urged the use of the binomial expansion expressions.

II DERIVATION OF ABSORPTION COEFFICIENTS

An absorption coefficient is generally defined by adding a velocity term to the equation of motion of an individual electron excited by a radio wave:

$$m_e\,(\ddot{\underline{r}} + \nu_{eff}\,\dot{\underline{r}}) = q_e \underline{E}_r + q_e \dot{\underline{r}} \times \underline{B}$$

The constant coefficient of the velocity term is seen to be the product of the mass of an electron and some effective collision frequency. This effective electron collision frequency is the weak link in the mathematical formalism describing the physics of the situation, since such a collision frequency must necessarily be dependent on velocity for an individual electron. In 1961 this subject was treated in detail by Sen and Wyller[15] and Cook and Lorents.[16] The effect of defining such a velocity-dependent term is to produce the Z term in Appleton's equations which manifests itself as χ, the imaginary part of the complex refractive index, n.

The defining equations are

$$n = \mu - i\chi \qquad E = E_0 e^{-\kappa h} \qquad \kappa = \frac{\omega\chi}{c} .$$

If $n = \mu - i\chi$, then a wave propagating vertically (in the h direction) attenuates as exp $\{-\kappa h\}$ where κ = the radian wave frequency times χ, the imaginary part of the refractive index, divided by c, the speed of light. The cumulative two-way absorption is given approximately by the expression

$$17.36 \int_0^{H=\text{Height of Reflection}} \kappa(h)dh \qquad \text{db}$$

where the factor 17.36 converts nepers to decibels. The cumulative two-way absorption referred to here must be added to the free-space divergence (inverse distance squared) to obtain the actual weakening of the power flux density of the original transmitted wave.* By equating $n^2 = \mu^2 - \chi^2 - 2i\mu\chi$ to the n^2 expression given by Booker for the QT approximation

* Whitehead points out that for such a calculation one must assume that the amplitude of the wave decays inversely as $\int ds/\mu$ and not inversely as $\int \mu' ds$ (the group path), as is occasionally erroneously assumed.[17] For an ionogram, $2h'$ is used as the total distance.

and equating imaginary parts, the following κ expressions are obtained:

$$\kappa_{\text{QT (upper)}} \cong \frac{\nu}{2\mu c} \cdot \frac{X}{(1 + Z^2)}$$

$$\kappa_{\text{QT (lower)}} \cong \frac{\nu}{2\mu c} \cdot \frac{X\left\{1 + \dfrac{Y_T^2}{[(1-X)^2 + Z^2]}\right\}}{\left\{1 - \dfrac{Y_T^2\,(1-X)}{[(1-X)^2 + Z^2]}\right\}^2 + Z^2\left\{1 + \dfrac{Y_T^2}{[(1-X)^2 + Z^2]}\right\}^2}$$

The upper sign corresponds to the case where the geomagnetic field is neglected. The lower-sign case is seen to be of the same form but more complicated. It is apparent that κ is basically a product of the collision frequency, ν, and the electron density divided by μc, which is, in the upper HF or VHF approximation ($Y << 1$, $Z << 1$), very nearly the group velocity modified by some function of X, Y, and Z, and θ.

By making the same substitution for the binomial expansion expressions for n^2, we get similar but more complicated expressions for κ:

$$\kappa_{\text{QT (upper)}} \cong \frac{\nu}{2\mu c} \cdot \sin^2\theta \cdot \frac{X}{(1 - X\cos^2\theta)^2 + Z^2}$$

$$\kappa_{\text{QT (lower)}} \cong \frac{-\nu}{2\mu c} \cdot \frac{\sin^2\theta}{\cos 2\theta}$$

$$\cdot \frac{X\left[1 - \dfrac{\sin^2\theta}{\cos 2\theta} \cdot \dfrac{Y_T^2}{(1-X)^2 + Z^2}\right]}{\left[1 - \dfrac{X\cos^2\theta}{\cos 2\theta} + \dfrac{\sin^2\theta}{\cos 2\theta} \cdot \dfrac{Y_T^2\,(1-X)}{(1-X)^2 + Z^2}\right]^2 + Z^2\left[1 - \dfrac{\sin^2\theta}{\cos 2\theta} \cdot \dfrac{Y_T^2}{(1-X)^2 + Z^2}\right]^2}.$$

The modifying expression is now a function of the angle, θ, which the wave normal makes with the earth's magnetic field; for $\theta = 90°$, the expression is identical with those derived from the n^2 expressions given by Booker. The minus sign in the expression for the extraordinary wave is canceled by the $\sin^2\theta/\cos 2\theta$ expression, which becomes -1 for $\theta = 90°$.

For both X and Z small, the lower-sign expression predicts a pole (high absorption) when the wave frequency equals the gyrofrequency; this is analogous to the more familiar longitudinal-propagation case.

For the case of nondeviative absorption, the differential absorption of the ordinary wave relative to the extraordinary wave can be calculated merely by taking a ratio of the κ expressions. When deviative absorption is significant, however, μ can no longer be assumed to equal unity and be canceled in such a ratio of κ expressions.

III ABSORPTION CALCULATIONS

In order to use these κ expressions to predict differential absorption, one should assume a model ionosphere and calculate the cumulative absorption given by the integral of κ with respect to height.

Figure 1 represents such a model ionosphere. In the middle of Fig. 1 is a curve of radian gyrofrequency in Thailand. A dipole field has been assumed, with the field decaying as r^{-3}. This corresponds to a gyrofrequency f_H of approximately 1 Mc at ionospheric heights in Thailand. This value is lower than that obtained at higher latitudes but is large for magnetic equatorial latitudes. On the right-hand side of Fig. 1 are the $N(h)$ profiles for day and night used for the calculations. These correspond to *foF*2 day and night values of roughly 9 and 2.8 Mc, respectively, which are typical of values observed in Thailand during the 1962-1963 sunspot minimum. On the left of the figure collision-frequency profiles for day and night appear. These curves correspond to collision-frequency profiles appropriate for the radio case as deduced by Cook and Lorents[16] using Boltzmann's equation and averaging over the important ionospheric constituents, assuming a Maxwellian velocity distribution.

By using the nighttime model ionosphere shown in Fig. 1 and the absorption coefficient expressions given for the bionomial expansion, reduced to Booker's expression for $\theta = 90°$, the $\kappa(h)$ curves shown in Fig. 2 were computed for 1, 2, 5, and 10 Mc.

It can be seen that 1-Mc reflections occur at true heights greater than 200 km and that 5 and 10 Mc penetrate. The effect of the magnetic field is seen to become less important as the wave frequency becomes appreciably greater than the gyrofrequency. When the wave frequency is six times the gyrofrequency, the effect of the field is insignificant. The dashed curves are for the extraordinary ray, the ray more affected by the field. The solid lines are for the ordinary wave (assuming the field is zero when $\theta = 90°$). The κ expression can be thought of, with a change of scale, as having dimensions in decibels/kilometer (1 neper/meter = 8.686×10^3 db/km). As previously observed, κ is approximately the product of collision frequency and electron density in nondeviative regions. This is quite evident by the peak in κ values for all frequencies in the

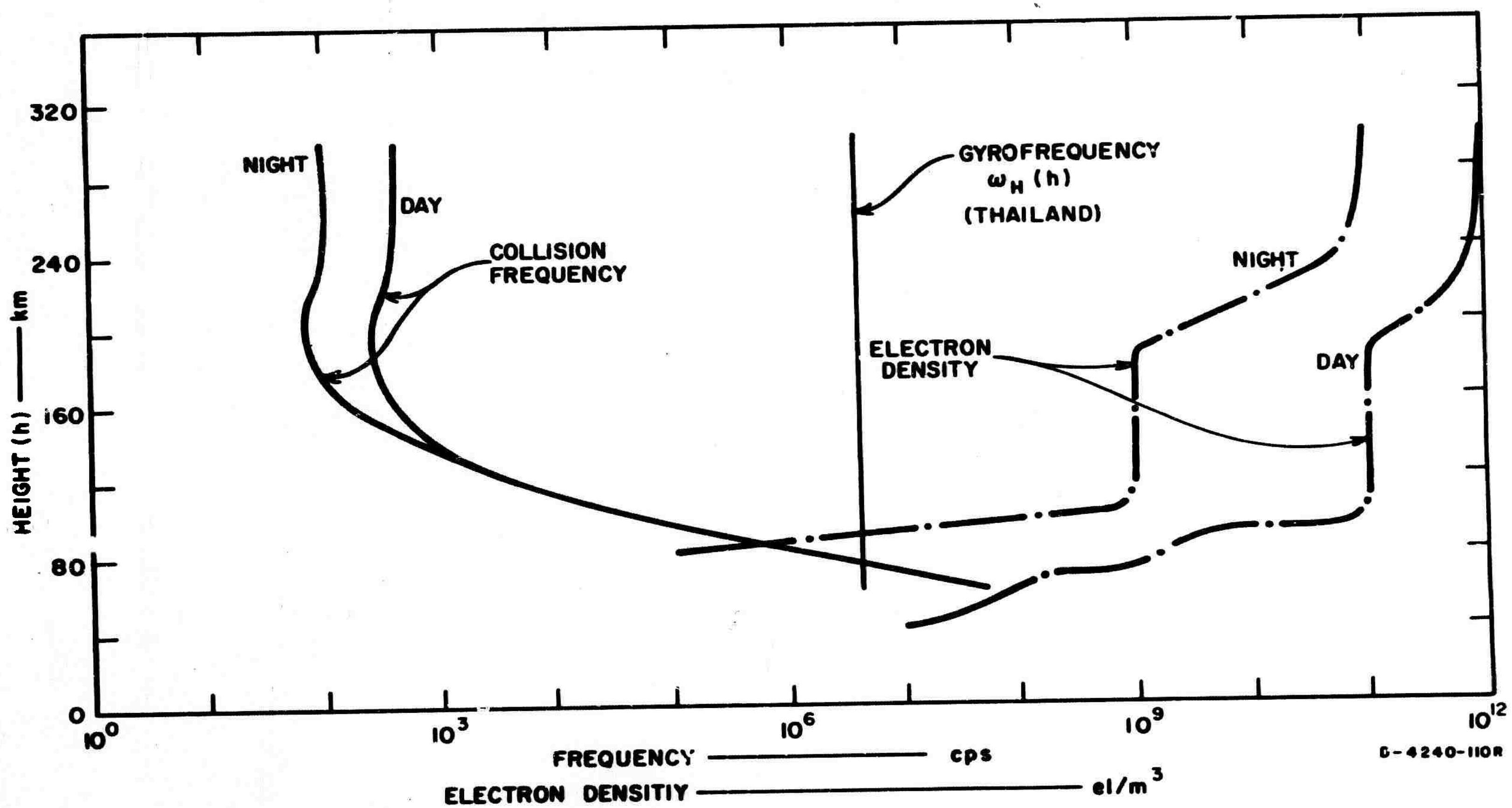


FIG. 1 MODEL IONOSPHERE

region near 110 km. Deviative absorption becomes important in the last few kilometers before reflection, as shown in Fig. 2. Notice that κ does not become infinite for reflection heights. This is because μ does not fall to 0 at reflection, an effect due to collisions. Reflections were assumed to take place at true heights where $X = 1$ and $X = 1 - Y$ for the ordinary and extraordinary waves.

These $\kappa(h)$ plots, especially in the deviative region near the top of the path, are heavily dependent upon the assumed $N(h)$ and $\nu(h)$ profiles shown in our model. These assumed profiles also contribute some of the irregularities shown in the curves (see Sec. IV-B for further information on assumed profiles). The important consideration, however, is the cumulative absorption; Fig. 3 shows this for the $\kappa(h)$ profiles of Fig. 2.

The ordinary wave at 1 Mc suffers less than 2-db absorption and that for the extraordinary is only about 14 db. The small hook at the top of each curve shows the deviative absorption occurring near reflection heights. (See Sec. IV-A for a more complete discussion of this point.) Most of this occurs within the last few kilometers before reflection, as pointed out by Whitehead in 1956.[18] For the nighttime case, at vertical incidence, at the magnetic equator, during sunspot minimum, the major contributor to total absorption is nondeviative and occurs at E-region heights for the profiles chosen. This is not necessarily true when the wave frequency is too near the penetration frequency for any of the layers or when scatter-type reflections (spread F or sporadic E) are present.

Figure 4 shows $\kappa(h)$ values for the daytime case, at vertical incidence, at the magnetic equator, during sunspot minimum. Notice the large values of κ near reflection for both O and X at 1 Mc. One neper/meter is almost 10 db/meter. It can be seen that the absorption coefficient for the extraordinary wave is always larger than that for the ordinary. For the electron density profiles assumed in Fig. 1, 1 Mc reflects in the upper D or lower E region; 5 Mc reflects at F-region heights; and 10 Mc barely penetrates the $F2$ layer which has $foF2$ of 9 Mc.

The valleys around 90 km are due to the product of relatively constant $N(h)$ profile and decreasing $\nu(h)$ profile when ions become important and electron density is still nearly constant with height. The valley at 210 km occurs because of the rapid increase in $N(h)$ and the relative constancy of ν with increasing height. The small valley at approximately

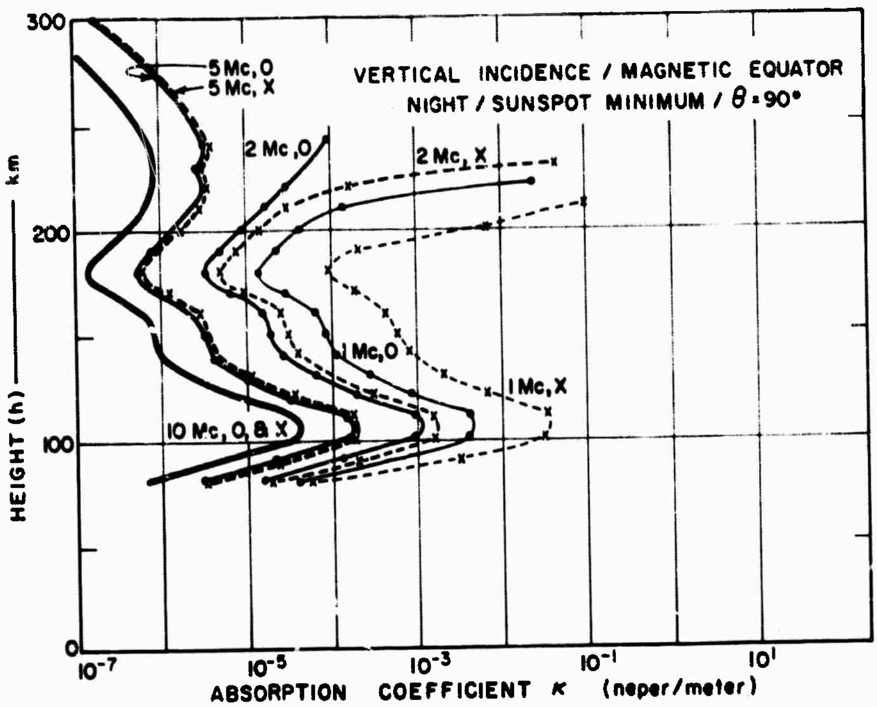


FIG. 2 ABSORPTION COEFFICIENT κ FOR NIGHT, SUNSPOT MINIMUM, $\theta = 90°$

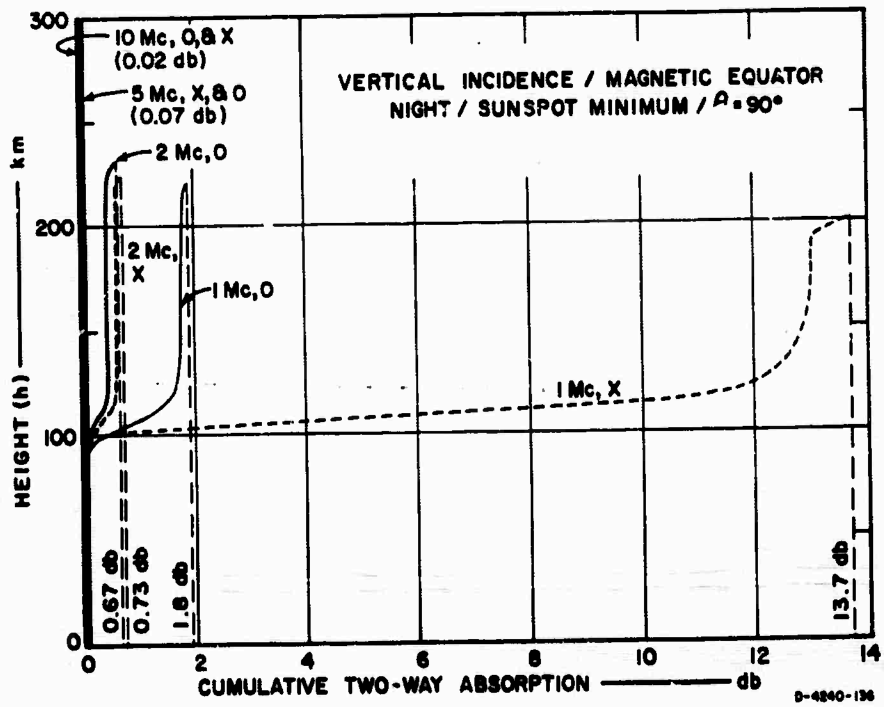


FIG. 3 CUMULATIVE TWO-WAY ABSORPTION FOR NIGHT, SUNSPOT MINIMUM, $\theta = 90°$

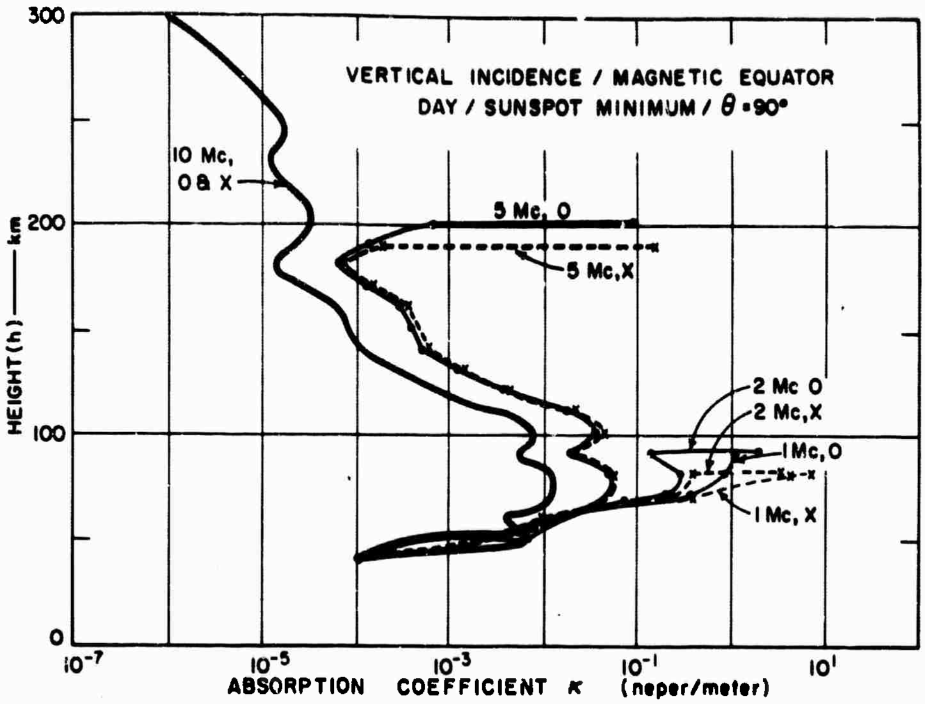


FIG. 4 ABSORPTION COEFFICIENT κ FOR DAY, SUNSPOT MINIMUM, θ = 90°

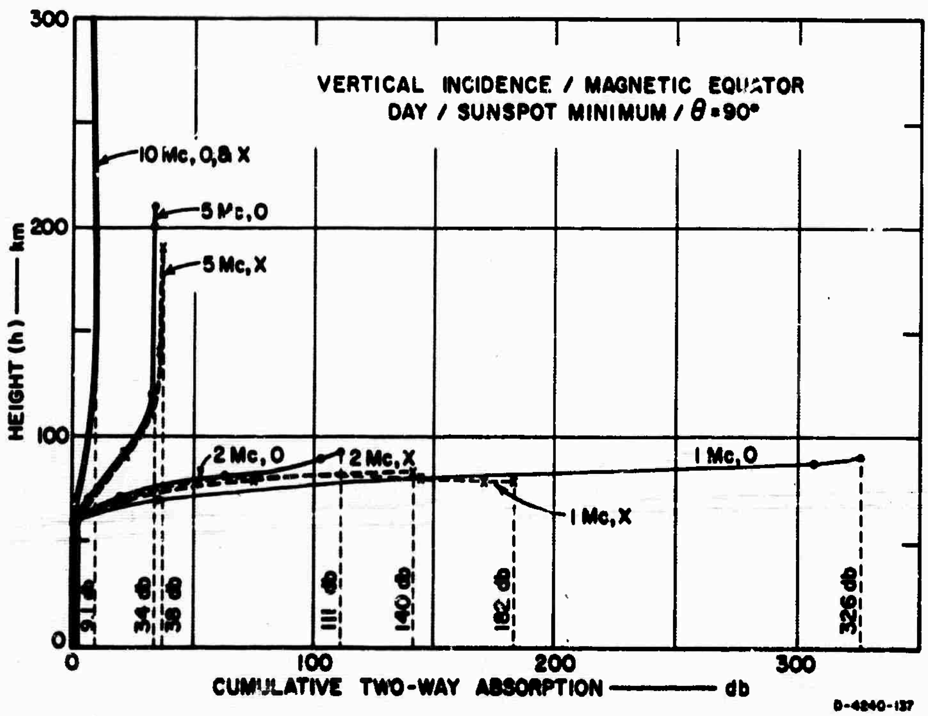


FIG. 5 CUMULATIVE TWO-WAY ABSORPTION FOR DAY, SUNSPOT MINIMUM, θ = 90°

60 km is perhaps associated with the height at which $Z = 1$. One would expect coupling at this height but ray theory does not predict this. The 60-km valley is due, therefore, to the profiles. This curve is basically similar to one obtained by Webber at high latitudes in regions where the QL approximation is valid.[19] (See Sec. V for a more complete discussion of the QL case.)

Figure 5 shows the cumulative absorption for the daytime case. It is apparent that even though the 1-Mc absorption coefficient for the extraordinary wave is everywhere greater than that for the ordinary, the ordinary wave suffers much greater total absorption. This is caused by deviative absorption in a region of high electron-density collision-frequency produce (the region normally responsible for nondeviative absorption of HF waves). Here, however, one would require a full-wave solution to accurately determine the total effective deviative absorption (aee Sec. IV-A). The 5-Mc daytime absorption is about twice the 1-Mc nighttime absorption. Again, most of the 5-Mc absorption occurs in the *D* and *E* regions. The 10-Mc wave picks up about 10 db in its hypothesized two-way passage through the region up to 120 km and suffera negligible absorption along the rest of its path. This wave actually penetrates the *F*2 layer, aa previoualy stated.

The cumulative abaorption curves may be more meaningful if we consider a comparison between the relative aignal loss due to increased distance when e higher frequency (reflecting at greater height) is uaed and the decreaaed ionospheric absorption due to collisions.

For the free-apace caae, uaing a power-flux-density approach,

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2} .$$

Assume $G_T = G_R = 1$ and define $d = 2h'$ (*i.e.*, assume virtual source). The equation becomes

$$\frac{P_T}{P_R} = \left(\frac{8\pi h'}{\lambda}\right)^2 = \left(\frac{8\pi h' f}{c}\right)^2 .$$

$$\text{Free-space (isotropic) path loss} = 10 \log_{10}\left(\frac{P_T}{P_R}\right) .$$

Table I

IONOSPHERIC PATH LOSS

LOCAL TIME (hr)	FREQUENCY (Mc)	h' (km)	ISOTROPIC FREE-SPACE LOSS (db)	IONOSPHERIC ABSORPTION (db)	TOTAL PATH LOSS (db)
1000	2	100	84.5	111	195.5
1000	5	350	103.3	34	137.3

One might ask: Since the curves presented thus far are for $\theta = 90°$, what is the effect of varying θ? The QT approximation holds in the HF range of frequencies only for a very small range of angles near $\theta = 90°$. In Thailand at 1 Mc, the QT approximation is valid upon entry to the ionosphere in the range of angles from approximately 9° to pure transverse; at 5 Mc, the QT holds over a range of angles less than 2° from $\theta = 90°$. The θ_T for entry in the lower ionosphere can be calculated so that the QT approximation holds for values of θ between θ_T and 90° by solving:

$$\theta_T = \text{arc sin} \left\{ -\frac{20}{Y^2} + \frac{1}{2}\left[\left(\frac{40}{Y^2}\right)^2 + 4\left(\frac{40}{Y^2}\right)\right]^{1/2}\right\}^{1/2} .$$

This assumes $Z \ll 1$, $Y \ll 1$.

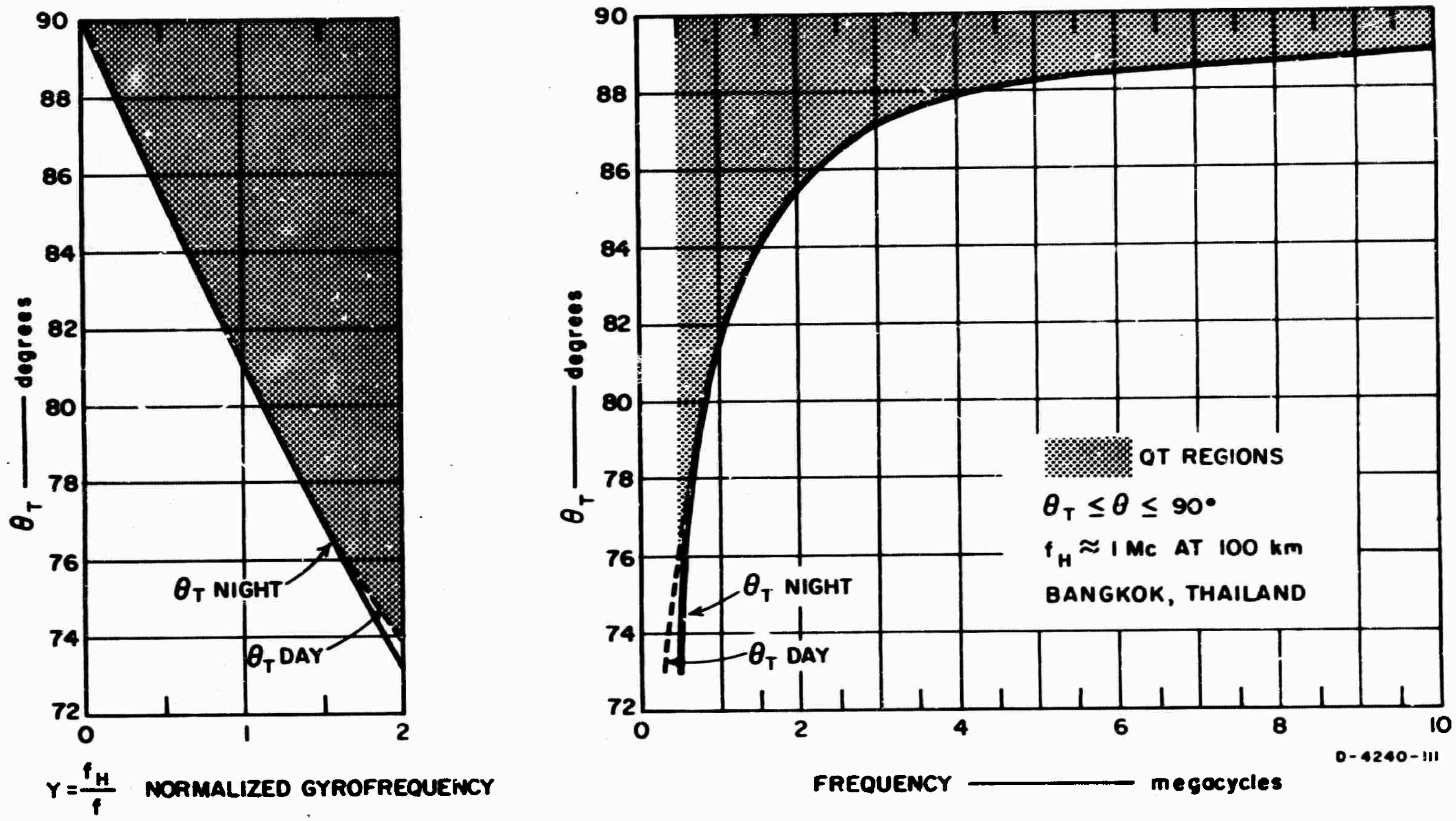
It can be shown that

$$\theta_T \cong \text{arc sin}\left[1 - \frac{Y^2}{40}\right]^{1/2}$$

for Y less than but not too near unity. For low frequencies, the approximation predicts too small: The error for $Y = 1$ is 13′ and the error for $Y = 1/2$ is only 2′. For $Y \ll 10$ the expression can be simplified to

$$\theta_T \cong \text{arc sin}\left(1 - \frac{Y^2}{80}\right) .$$

The exact relationship between θ_T and Y is given in Fig. 6.



(a) θ_T vs Y

(b) θ_T vs FREQUENCY

FIG. 6 GEOMETRICAL ASPECTS OF QT APPROXIMATION

An inspection of the κ expressions reveals that the trigonometric functions change very little over the range for which the QT approximation is valid at HF. The absorption values calculated for $\theta \approx 90^\circ$ appear to be significant indicators for the region where the QT approximation holds in the HF part of the spectrum in the ionosphere over Thailand. These assumptions were checked with calculations and demonstrated to be effectively true.

When the major portion of the absorption is nondeviative, vertical-incidence values can be converted by using a secant law normally referred to as Martyn's absorption theorem.[20] Geometrical and equivalent-path considerations are employed (i.e., magnetic-field effects are not included).

The previously mentioned calculations indicate that a few relatively small height ranges—for the operating frequency not too near the critical frequency of a layer—are the major contributors to the cumulative integral. This suggests that a graphical solution might be useful in estimating absorption. A few values of κ could be calculated [given an $N(h)$ profile]* by using the previous curves as a model and by plotting on a linear scale. The resulting area could be planimetered and multiplied by the appropriate factor to give approximate absorption in decibels.

---

* A reasonable true height profile could be obtained from an ionogram without the aid of a large computer by using the methods of Schmerling[21] or Heubert.[22]

IV DISCUSSION OF ERROR

A FORMULAS

The formulas used to calculate the total cumulative absorption suffer from several inadequacies:

(1) The formulas assume that the ordinary and extraordinary waves can be described separately (*i.e.*, no coupling)* for the entire trajectory. This assumption is most likely to fail at the base of the ionosphere (height determining limiting polarization)[23] and near reflection heights, especially for angles θ where the QT approximation is about to fail.

(2) There are other difficulties near the top of the trajectory.

(a) The formulas assume that ray-theory (*WKB*) solutions[25] apply for the entire trajectory. This is not necessarily true near the top of the trajectory, where the wavelength in the medium becomes very long, and a full solution of the differential equation is required.

(b) The equation for vertical incidence consists of integrating the imaginary part of the refractive index up to the "true height" of reflection, the height for which the refractive index in the medium would go to zero, neglecting collisions. It is not clear that this is the appropriate upper limit when the collision frequency is not negligible relative to the wave frequency near the reflection height. The refractive index does not go to zero at the top of the trajectory for vertical incidence in a horizontally stratified ionosphere when collisions are significant. Thus the reflection is not total; also, some of the energy that would otherwise have been stored in the evanescent wave above the true height is lost to the wave. A study of the full-wave solution in the region of reflection indicated (for a different location and consequently a different dip angle) that merely integrating the absorption coefficient to the true height of reflection predicts too little absorption; error could be as high as 10 to 15 db at 2 Mc.[26] This problem is further complicated when the wave frequency is near the critical frequency of any of the layers [*i.e.*, the $N(h)$ profile is very steep].

(c) Energy might be lost out the top of an "overdense" layer owing to the finite decay distance of the evanescent wave, even if collisions are negligible.

* See Budden[23] for a discussion of Foersterling's[24] work. See also Cohen who discusses coupling and the QT.[8]

(d) Except for exactly tranverse propagation at vertical incidence, the trajectory may be deviated near the top, because of the earth's static magnetic field. Also, horizontal gradients (tilts) or other irregular stratification may so alter the trajectory that the path integrals are not truly along the ray path.

(3) The *WKB* solutions may not be appropriate below the top of the trajectory (*e.g.*, sporadic *E*, and spread *F*).

(4) Cos θ changes sign for $\theta \neq 90°$ in down-going wave relative to up-going, so separate calculations are required for the up and down portions, but this does not appear important for the HF case. (See Crombie for a discussion of the VLF case.[27])

The assumption of no coupling in the lower ionosphere is probably the most appropriate at HF for vertical incidence with horizontal static magnetic field,[28] except when sporadic *E* is present. The difficulties near the top of the path are discussed by Fejer and Vice who use a full-wave solution to show that at 2 Mc a correction is required.[26] They conclude that more than half the total absorption occurs in the last 2 km before the "true height" of reflection (reflection in the lower *E* region). This reiterates Whitehead's conclusion and re-emphasizes that the slope of the electron density profile in the region of reflection is very important in determining deviative absorption. Also, the distance increment must be decreased in computing the value of the cumulative absorption integral from the value most economically used in the nondeviating region. We need to look more closely at the full-wave solution for the curves in this report, but the other possible difficulties need not concern us here.

Near reflection heights at vertical incidence, the *WKB* solutions are no longer appropriate—ray theory does not apply—and, when collisions are not negligible, the reflection coefficient can be shown to depend upon the properties of the Airy integral function.[23] This can be converted to a phase integral in the complex height (above the ground plane) to obtain the appropriate value of reflection loss or a full-wave solution can be performed (solution of Maxwell's equation in the medium under appropriate boundary conditions). By either technique it becomes apparent that the slope of both $N(h)$ and $Z(h)$ in the region near reflection without collisions (*i.e.*, $X = 1$ for the ordinary wave and $X = 1 - Y$ for the extraordinary) is very important.

Carefully calibrated experiments could resolve these difficulties and provide useful data for LUF calculations under various conditions.

Fortunately, this top-of-the-path difficulty is rarely encountered on oblique paths, where deviative absorption is often negligible at HF.

B. ASSUMED PROFILES

1. Comparison of Collision Frequency Profile Used for These Calculations with Measured Values

The collision frequency profiles of Cook and Lorents[16] were used for these calculations. Figure 7 shows these values for day and night plotted with other theoretical curves and some measured data.

On the far left (dotted curve), the particle collision frequency is given for reference. These values are much lower than those of the effective electron collision frequency because the average particle mass is much greater than the mass of an electron giving lower average velocities. Also, the relative interaction range for electrons is greater than that for neutral particles, causing the effective particle collision frequency to be lower.* (This curve, occasionally confused with the effective collision frequency because of ambiguity in the literature, is given here for comparison.)

First, notice Nicolet's 1953 values, which represent pioneering work in this field.[29] These values, now generally assumed to be too large by a factor of about 3 or 4, are included for comparison. Theoretical values given by Cook and Lorents[16] and experimental values given by Webber[19] (ARDC values processed using Fejer's measurement[30]) and Kane[31] are in excellent agreement in the *D* region. Up to about 130 or 140 km (through the *D* and *E* regions), the main constituents are generally taken to be N_2 and O_2. At about 200 km (day) or 250 km (night), ions tend to predominate, causing the knee in the $\nu(h)$ profiles as given by Cook and Lorents.[16] The collision frequency is least well-known in the *F* region and is assumed constant, with height above about 250 km. Fortunately it is less important to know the collision frequency accurately above 250 km for cumulative absorption calculations than that in the *D* and *E* regions, since the magnitude of the electron-density collision-frequency product is already quite small. Recent measurements indicate that the *F*-region collision frequency may be larger than was previously supposed (see Fig. 7). However, the assumed profiles of Cook and Lorents[16] represent the best available at the time of these calculations and are in reasonable agreement with measured data.

* Dr. R. C. Whitten, SRI, private communication (5 May 1964).

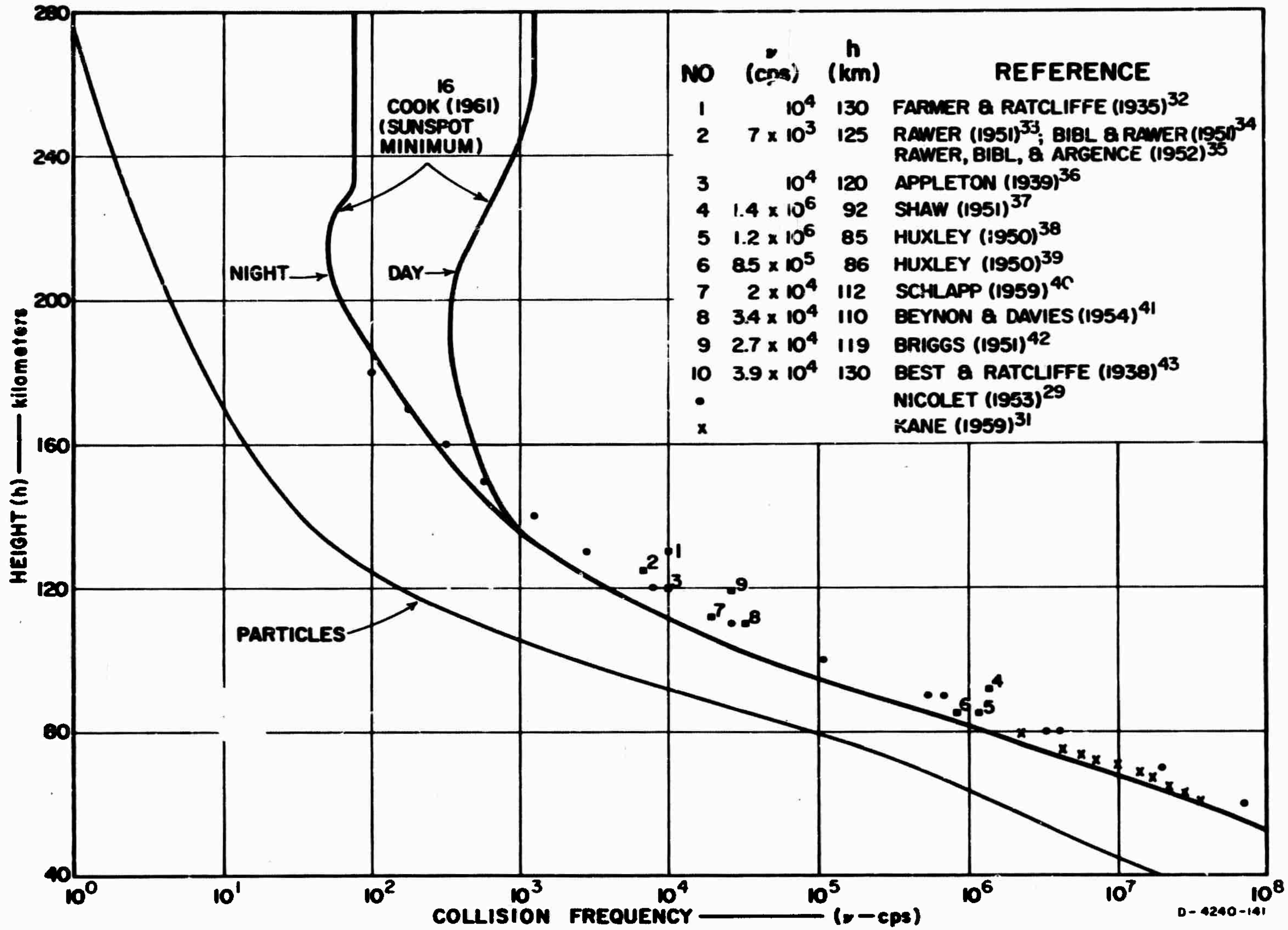


FIG. 7 COLLISION FREQUENCY PROFILES

2. Comparison of Electron Density Profiles Used for These Calculations with True Height Profiles Reduced from Bangkok Ionograms

A check on the assumed electron density profiles was obtained by making true height reductions, using Budden's method,[23] of typical day and night ionograms from Bangkok, Thailand. Unfortunately, these ionograms were not yet available when the original calculations were made. Figures 8 and 9 show the assumed and measured profiles for night and day, respectively. Observe that one would have to use a measured true height profile to relate measured amplitude data to absorption calculated by using the coefficients presented in this report. Nevertheless, the profiles assumed for the calculations in this report follow the general trend of the $N(h)$ profiles measured at Bangkok, Thailand (magnetic dip angle = 10°) day and night during sunspot minimum.

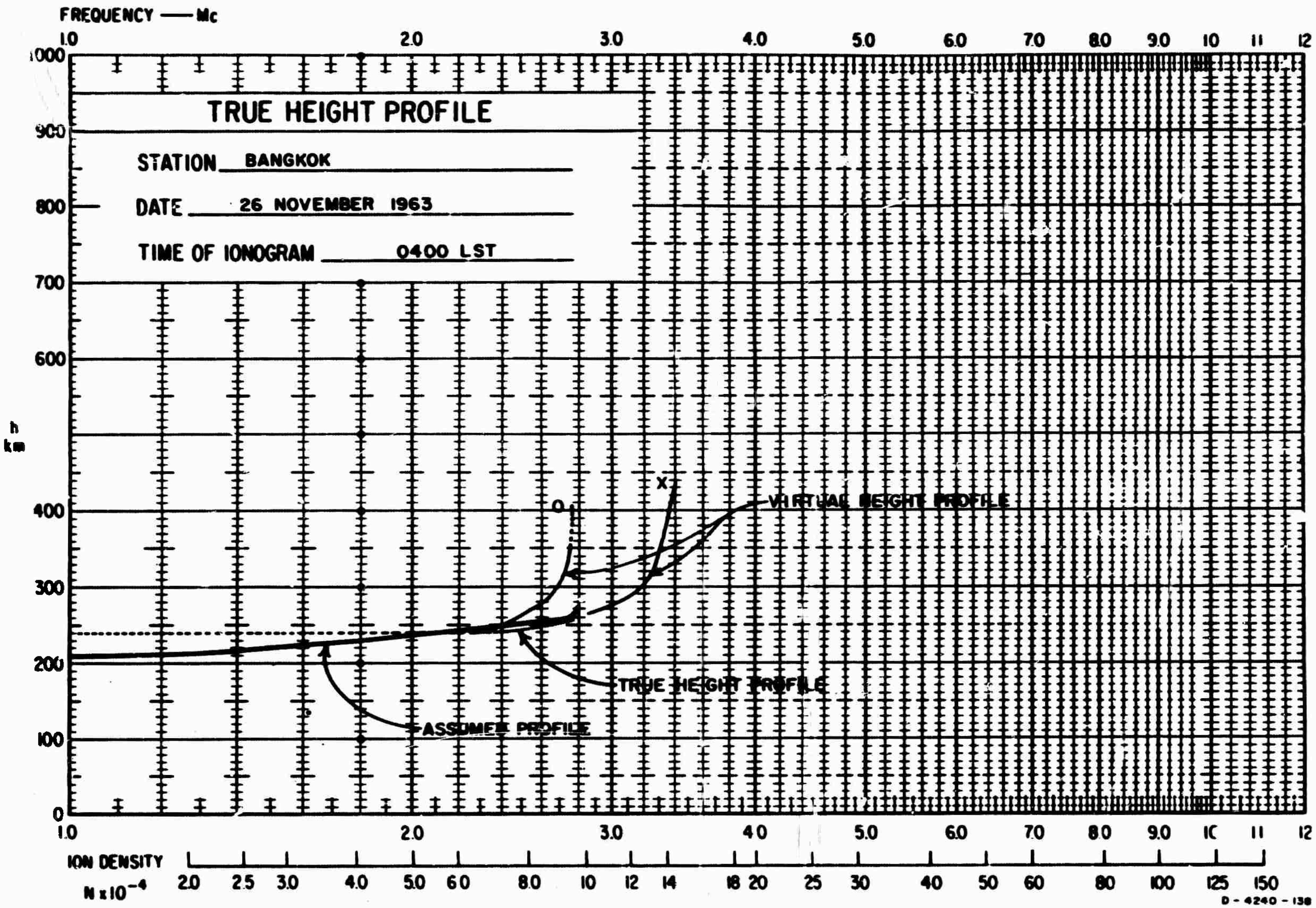


FIG. 8 ELECTRON DENSITY PROFILE — NIGHT

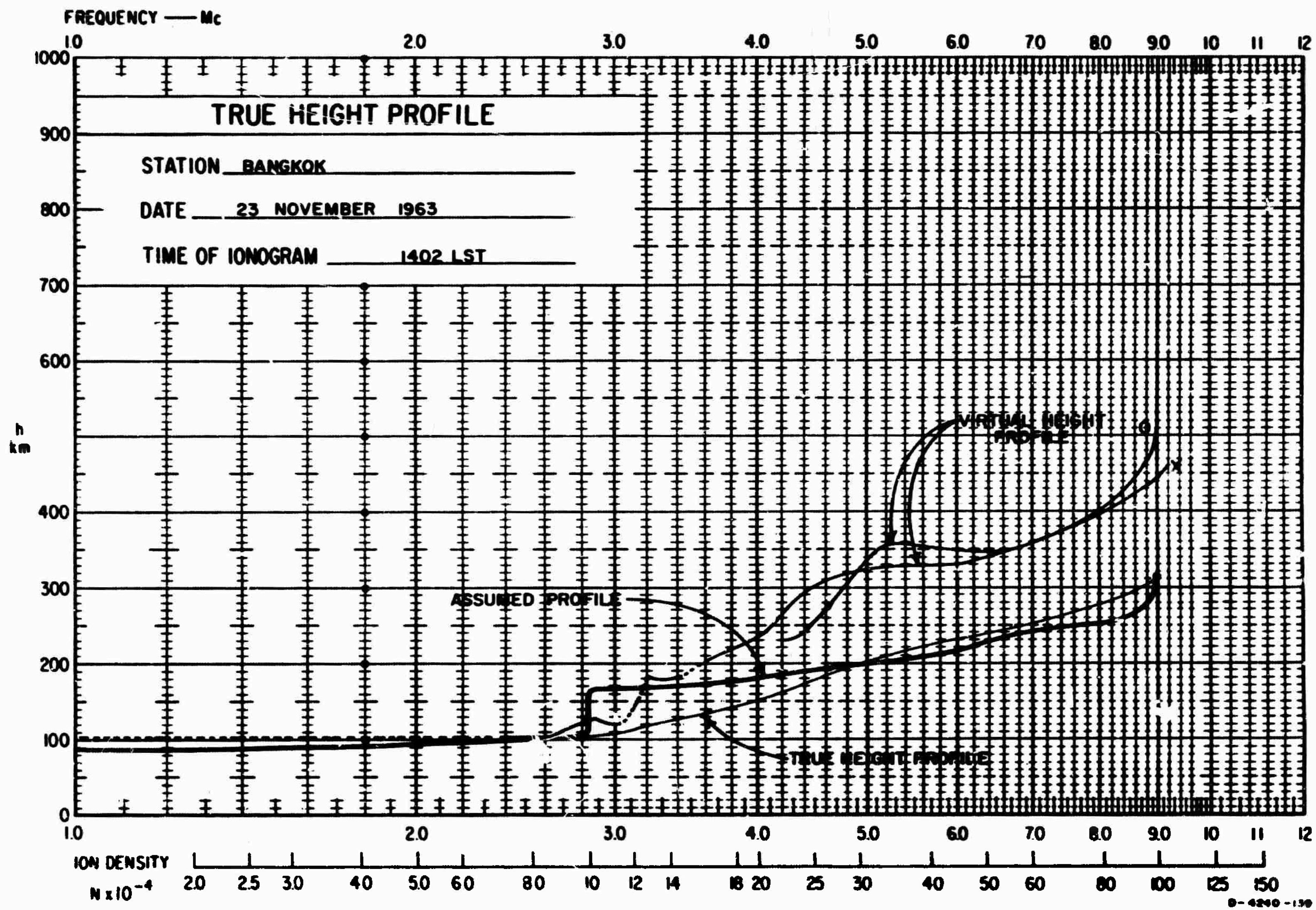


FIG. 9 ELECTRON DENSITY PROFILE — DAY

V COMPARISON OF QL AND QT ABSORPTION VALUES

The most commonly used and widely applicable simplification of Appleton's equation is the quasi-longitudinal (QL) approximation:[9]

$$\frac{Y_T^4}{4Y_L^2} \ll \left|(1 - X - iZ)^2\right| \quad .$$

The resulting simplifications in refractive index expressions are

$$n^2_{QL\ (upper)} \cong 1 - \frac{X}{1 - iZ + |Y_L|}$$

$$n^2_{QL\ (lower)} \cong 1 - \frac{X}{1 - iZ - |Y_L|}$$

with corresponding absorption coefficients

$$\kappa_{QL\ (upper)} \cong \frac{\nu}{2\mu c} \cdot \frac{X}{(1 + |Y_L|)^2 + Z^2}$$

$$\kappa_{QL\ (lower)} \cong \frac{\nu}{2\mu c} \cdot \frac{X}{(1 - |Y_L|)^2 + Z^2} \quad .$$

At vertical incidence, these formulas are most applicable in polar regions and in temperature regions at higher frequencies ($f > 5$ Mc). The approximation should, of course, be checked for individual cases. Characteristic waves are circularly polarized in the QL approximation:

$$R = \mp i \quad .$$

To illustrate the difference between this more familiar case and the QT, calculations were made of absorption coefficients and cumulative absorption with the same model ionosphere (Fig. 1) but with a purely longitudinal magnetic field. The results are shown in Figs. 10 through 13.

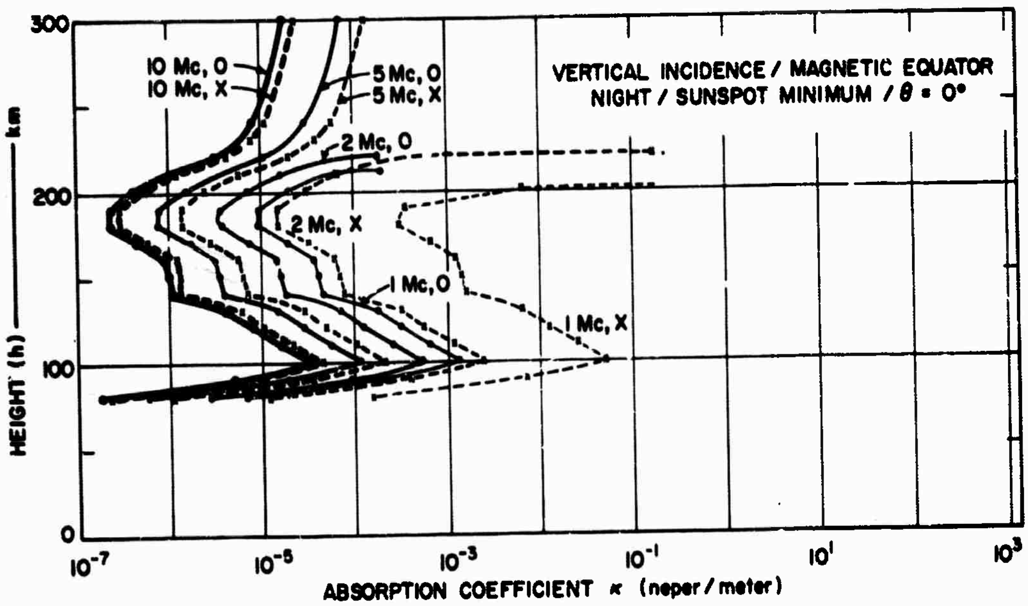


FIG. 10 ABSORPTION COEFFICIENT κ FOR NIGHT, SUNSPOT MINIMUM, θ = 0

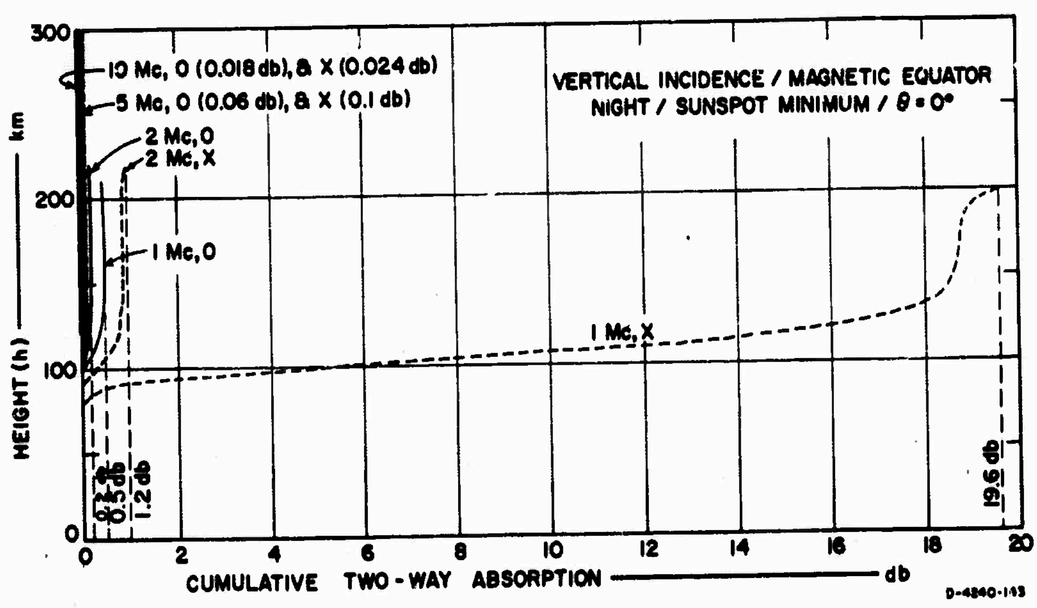


FIG. 11 CUMULATIVE TWO-WAY ABSORPTION FOR NIGHT, SUNSPOT MINIMUM, θ = 0

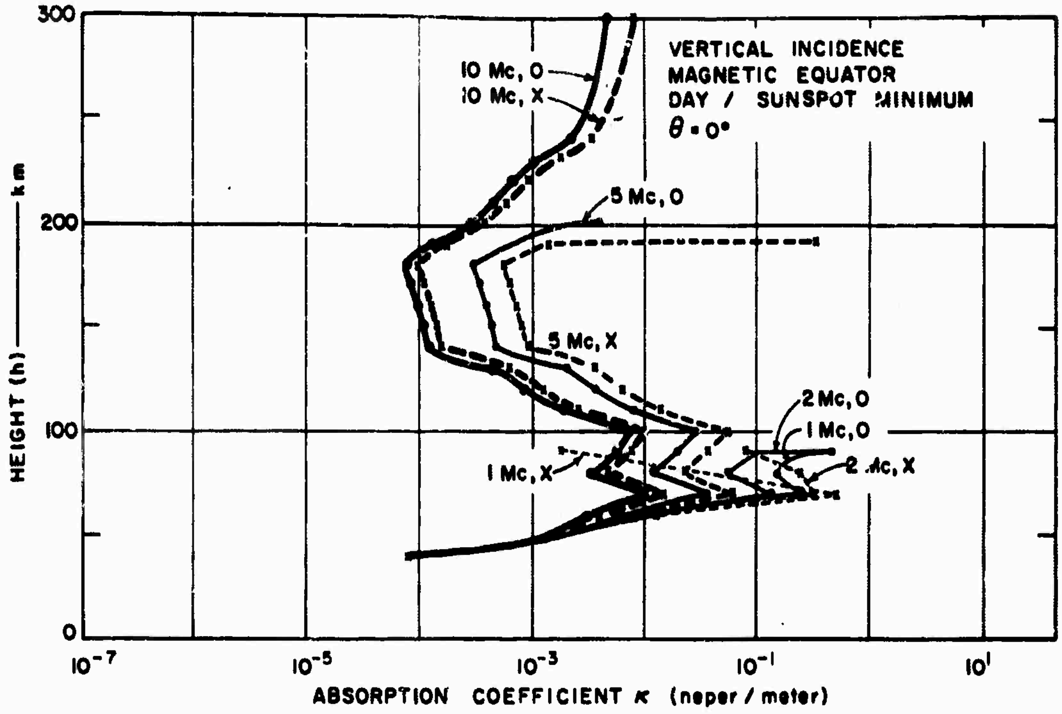


FIG. 12 ABSORPTION COEFFICIENT κ FOR DAY, SUNSPOT MINIMUM, $\theta = 0°$

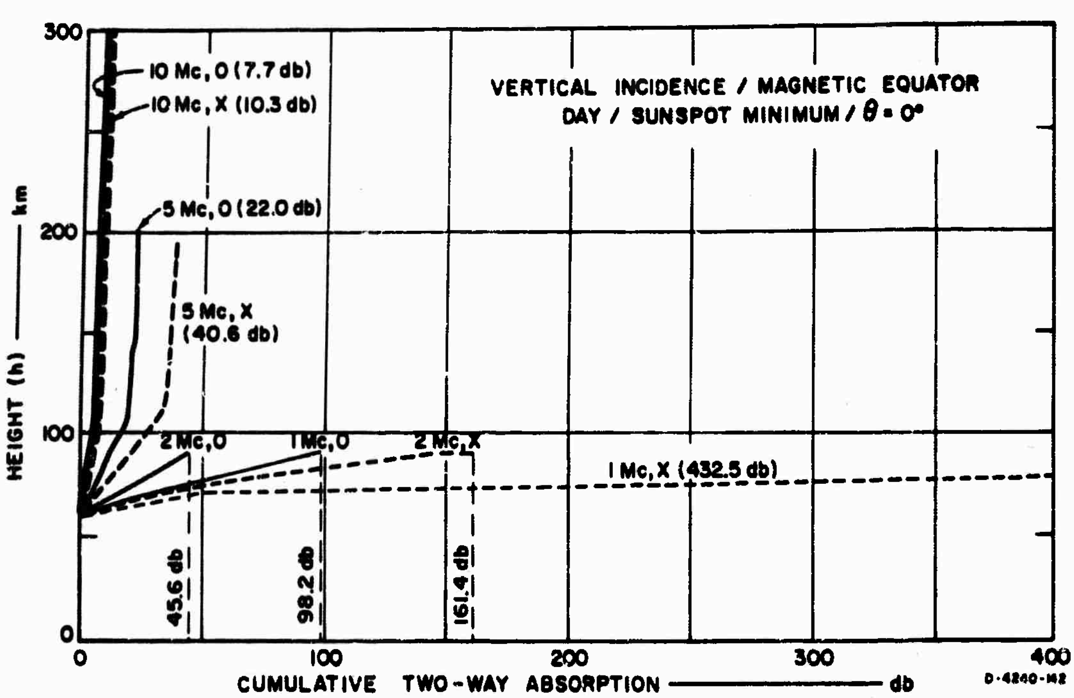


FIG. 13 CUMULATIVE TWO-WAY ABSORPTION FOR DAY, SUNSPOT MINIMUM, $\theta = 0°$

Figure 14 shows the 1-Mc κ-versus-h curves for night, QL and QT. Notice the greater spread between O and X absorption for the QL. Recall that the QT ordinary wave approximates the no-field case. The QL ordinary wave predicts less absorption than the no-field case (QT ordinary), whereas the QL extraordinary predicts more absorption than the QT extraordinary. The most striking feature of the QL, then, is the greater differential absorption; this is most easily observed on the 2- and 5-Mc data for day. Thus, one would expect polarization fading of a continuous wave at vertical incidence to be potentially worse in Bangkok, Thailand, than in Menlo Park, California, with linearly polarized (*e.g.*, dipole) antennas oriented at 45° to the earth's static magnetic field.

Piggott presents a technique for calculating absorption at low latitudes.[44] Since this formula includes the QL function $y(f,f_L) = [1/(f \pm f_L)^2]$ for $f^2 >> f_L^2$ (positive sign for the ordinary and negative sign for the extraordinary) rather than the appropriate QT functions, it appears to the author that Piggott's formula will predict too little absorption for the ordinary and too much for the extraordinary even when f_L is replaced by f_T for the extraordinary and zero for the ordinary. For pure transverse, $f_L \triangleq 0$ and the function $y(f,f_L)$ would be $1/f^2$, which is surely not the case for the extraordinary. Piggott adds constants of approximately 4 db at 2 Mc and 10 db near $foF2$, on the basis of experimental data at vertical incidence. The author suggests that when accurate values are required in regions where the QT approximation is valid, the techniques presented in this report (when checked and modified by experimental data to account for difficulties at the top of the ray trajectory) provide a better method of calculation for the vertical- and near-vertical-incidence case. The formulas given in this report, modified by experimental data, could be used to generate nomographs similar to those in Piggott's report. The author concurs with Piggott's practical approach of considering frequencies near layer critical frequencies not useful for communicating. At vertical incidence, there is need not only for information on LUF for a given system and $foF2$ but also for knowledge of the unusable bands near foE and $foF1$.

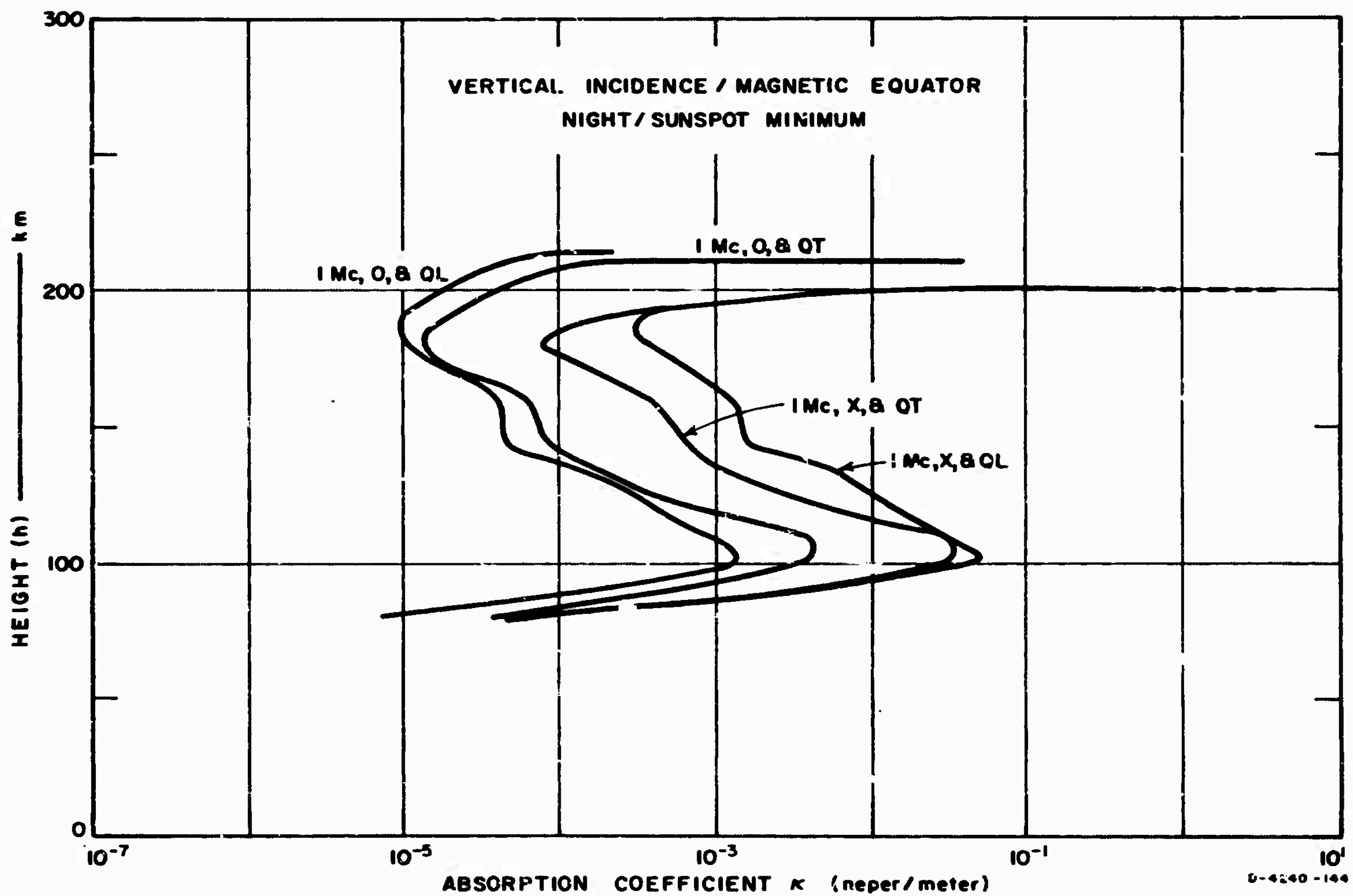


FIG. 14 NIGHTTIME κ vs. h FOR QL AND QT

VI SUMMARY AND CONCLUSIONS

In summary, the absorption coefficient expressions for the QT case (bionomial expansion) have been presented. The QT approximation is most likely to be valid for short equatorial paths, long east-west paths, parts of some north-south paths at high latitudes, reflections from field-aligned ionization (where reflection takes place transverse to the field), and near-reflection heights for the ordinary wave in the region where the *WKB* solutions still apply. Limitations are that the QT approximation must apply (a geometrical condition, see Fig. 6), ray tracing is appropriate, there is no significant coupling between O and X, and the models chosen for $N(h)$ $\nu(h)$ are appropriate. Some of these assumptions may not be valid at vertical incidence near reflection heights. Lacking substantiating measurements, the author has greater confidence in the calculation of nondeviative absorption than in that of deviative absorption. The full-wave solution should be adequate to estimate deviative absorption, provided the wave frequency is not too near the critical frequency of a layer. In the latter case, the deviative absorption would probably be so great as to make a surrounding band of frequencies unusable and absorption calculations an academic matter.

Conclusions of this study pertinent to ionospheric propagation in Thailand are in agreement with the scant available empirical data:[45] that an ionospheric hop is generally much better than a ground wave for distances exceeding about one mile in jungle. Pulse (50-μsec) tests between Bangkok and Pak Chong, Thailand during daytime using a C-2 sounder (approximately 10-kw) in October 1963, showed no reception on 2 Mc, but pulses were received on 2.5 Mc and above.* The test was conducted at about 10 a.m. local time. The cumulative absorption (two-way) calculations presented in this report indicate that the ionospheric absorption could account for the lack of reception on 2 Mc and the reception of 2.5 Mc during daytime on that 140-km path. From ionospheric absorption considerations of the regular layers, one would want to try to excite the ordinary wave on a frequency below but not too near foE or $foF2$ when using low-power (manpack) HF sets. This can be done by aligning a linearly polarized antenna parallel to the earth's magnetic field for both transmitting and receiving, regardless of

* C. W. Bergman, private communication (22 November 1963).

the path direction, provided that the path is short (less than several hundred kilometers). Differential absorption may provide some "mode purification" and produce less severe fading for combinations of parameters where the ordinary wave is less absorbed than the extraordinary, even though some extraordinary wave has been generated; however, differential absorption is not so great as in the QL case.

Experiments similar to those of Whitehead[17] could be performed in Thailand to test the applicability of the absorption coefficients derived in this paper. Measurements would also give better definition of the useful frequency range for low-power radio sets designed for short-path communication via the ionosphere in an equatorial jungle environment. Such experiments should check the error in deviative absorption calculations due to the uncertainty in true height of reflection (especially for the lower frequencies in the HF band during day), define how close to *foE* and *foF*1 one can successfully operate with low-power (≈1 watt) sets, and provide data for making absorption nomographs for near-verticle-incidence paths, involving local time, sunspot number, operating frequency, etc. These nomographs together with reliable noise data would provide a means for making LUF calculations for a given type of radio set, time, and place (see Appendix B).

APPENDIX A.

NOMENCLATURE

APPENDIX A

NOMENCLATURE

c = the velocity of light in vacuum

$\vec{B}$ = the magnetic flux density vector of earth's static magnetic field

q_e = the electronic charge (1.6×10^{-19} coulomb)

$\dot{\vec{r}}$ = the velocity vector of an individual electron

$\ddot{\vec{r}}$ = the acceleration of an individual electron

ν = the effective electron collision frequency

N = the electron density

h = the altitude above mean sea level

h' = the virtual height of reflection

R = the radius of earth ($\approx$6367 km)

f = the operating frequency

f_H = $\omega_H/2\pi$ = the electron gyrofrequency

f_p = $\omega_p/2\pi$ = the electron plasma frequency [f_p^2 = (constant)(N)]

$foF2$ = the critical frequency of the $F2$ layer for the ordinary ray

f_c = the layer critical frequency

n = the complex refractive index of the ionosphere

μ = the real part of the refractive index

χ = the imaginary part of the refractive index

X = ω_p^2/ω^2 the normalized plasma frequency squared

Y = ω_H/ω the normalized gyrofrequency

Z = ν/ω = the normalized collision

θ = the angle between the static geomagnetic field and the wave normal

Y_L = $Y \cos\theta$ = the longitudinal component of the normalized gyrofrequency

NOMENCLATURE

Y_T = $Y \sin \theta$ = the transverse component of the normalized gyrofrequency

ω_p^2 = $N_e^2/\epsilon_0 m$ = radian plasma frequency squared

e = the electronic charge

ϵ_0 = the electric permittivity of free space

m_e = the electronic mass

ω_H = $\mu_0 H_0 e/m$ = radian gyrofrequency

μ_0 = the magnetic permeability of free space

H_0 = the magnetic intensity of the space static geomagnetic field

i^2 = -1

APPENDIX B

APPLICATION OF ABSORPTION COEFFICIENTS TO LUF CALCULATION

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APPLICATION OF ABSORPTION COEFFICIENTS TO LUF CALCULATION

The lowest useful frequency (LUF) for a given system (employing an ionospheric reflection), time, and location is determined by considering a complex combination of parameters. Details of the system must be known, as well as propagation conditions and the noise environment. Generally speaking, the lower the frequency the greater the absorption and hence the smaller the signal. Also, at the lower frequencies the atmospheric noise is greater, increasing the required signal for a given $(S + N)/N$. The calculation of LUF is basically that of determining the lowest frequency that gives a "usable" signal; this implies a value judgment on the required $(S + N)/N$ ratio and fading margin (*i.e.*, allowable error rate).

The LUF might be defined as the frequency at which the received signal is equal to the required signal (see Fig. B-1).[46]

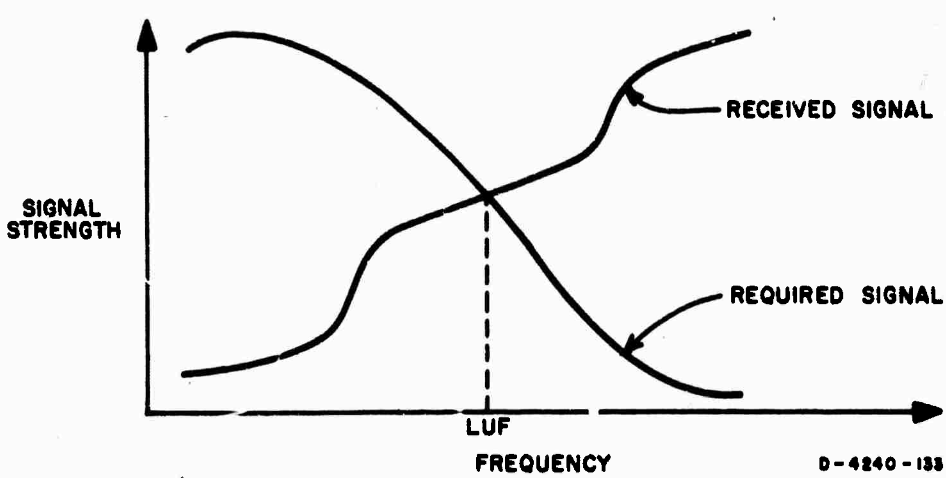

FIG. B-1 LUF DEFINITION

This report has considered absorption, which affects both curves. An example of the calculation of the received signal curve for near-vertical incidence is given in Sec. III, using the assumed profiles discussed in Sec. IV-5 for day (1000 local time). For an actual system, the antenna gains relative to isotropic (in db) should be subtracted from the total path loss to obtain the effective path loss. This is then subtracted from the transmitter power (in decibels referred to 1mw) to obtain the received signal power. This completes the received signal calculation.

Determination of the required $(S + N)/N$ is dependent upon the type of signal (modulation) and the type of noise. This is treated in numerous Signal Corps reports.[46-48] The external noise (atmospheric, man-made, or cosmic) is determined from noise maps[49,50] and added to the set noise.* Appropriate margins for both the noise deviations and signal fading are determined for the modulation and type of service used. The required received signal power to achieve the specified $(S + N)/N$ is calculated and plotted on the same scale versus frequency as the received signal. As previously stated, the intersection of these two curves determines the LUF.

* Note of caution: The noise values given on the maps[49,50] are for "the atmospheric noise power which a short vertical antenna abstracts from the surroundings and delivers to a matched receiver at a frequency of 1 Mc." While curves are given to extrapolate to other frequencies, when other antennas are used (e.g., horizontal dipole a quarter wavelength above ground) an assumption is required to convert from vertical to horizontal polarization and from isotropic azimuthal pattern to the required pattern (e.g., dipole). This subject is discussed briefly in Research Memorandum 5 Revised.[2]

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